FIELD EMISSION CURRENT OSCILLATIONS IN METALS LOCATED IN A MAGNETIC FIELD

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Quantum oscillations of the field emission current in metals located in a magnetic field perpendicular to the emitting surface are investigated theoretically for an arbitrary electron dispersion law. It is shown that, in addition to ordinary oscillations of the de Haas-van Alphen type and oscillations related to the chemical potential, there may also exist a peculiar type of oscillation which does not depend on temperature (except for dependence through the Dingle factor). A correction is made to Eq. (14) for ordinary oscillations in ref. 1.

IN an earlier paper[1] we investigated quantum oscillations of the field emission current of metals in a magnetic field perpendicular to the metal surface. We calculated oscillating terms of two types: 1) the ordinary type produced directly by oscillations of the number of electron states, and 2) terms produced by oscillations of the chemical potential. In the calculation the dependence of the transmission coefficient D of the barrier at the metal boundary on the quasimomentum p was not specified, and for estimates we used the free-electron model.

The present work utilizes the function D(p) obtained by us subsequently,[2] which permits us to complete the discussion of oscillations for an ordinary dispersion law. In addition, we take into account in the present article that contributions to oscillations of the emission current are received only from electrons which can complete many revolutions before the collision with the surface, i.e., electrons which arrive from deep inside the metal (see the note added in proof to ref. 1), and on this basis we have corrected Eq. (14) of ref. 1, which expresses the oscillating term of the first type in one of the cases. We have also found a new type of oscillation associated with the possible existence in the effective transmission coefficient (in the extremal orbit of the constant-energy surface) of a sharp maximum at an energy below the Fermi level. Although at zero temperature, oscillations of this type usually (but not always) should be smaller than those of the first type, in view of their weaker temperature dependence (only in terms of the electron-phonon collision time, which enters into the Dingle factor) they present considerable interest.

We note that these results are not contained in other published articles[3] on field emission in a magnetic field.

1. CALCULATION OF FIELD EMISSION CURRENT IN A MAGNETIC FIELD

Let a metallic single crystal, to which is applied a strong electric field which extracts electrons, be placed also in a magnetic field. The metal fills the half-space $z < 0$, and both fields are uniform and directed along the z axis. In this case the electron states in the metal are determined by the spin projection $\sigma = \pm 1/2$, the quantum number $n = 1, 2, 3, \ldots$, by the projections of the quasimomenta p_z and p_x (for choice of the vector potential in the form $A_x = -Hy$, $A_y = A_z = 0$), and also by the number of the group[1]. The emission current density is

$$j_z = -\frac{e^2H}{h^2c}\sum_{\sigma}\int\sum_{n}(i_z)_{np_z} f(E_n(p_z)+\sigma\mu_0 H)\,dp_z. \qquad (1)$$

Here we have already performed the integration over the variable p_x, on which the energy does not depend[2]; $(i_z)_{np_z}$ denotes the averaged z component of the probability flux density in the state n, p_z, corresponding to a normalization of the wave function to one electron per unit volume; f is the Fermi distribution function; $E_n(p_z)$ is the energy of orbital motion of the electron; $\mu_0 = e\hbar/m_0c$ is twice the Bohr magneton; if several different orbits correspond to given n, p_z, they all should be included in the sum over n.

The further calculations will be made on the assumption that the emission current is determined mainly by electrons whose states can be described quasiclassically ($n \gg 1$). Then the quantized levels $E_n(p_z)$ are determined from the relation[6]

$$S(E,\ p_z) = c^{-1}ehH(n+1/2), \qquad (2)$$

where $S(E, p_z)$ is the area of the section of the constant-energy surface $\mathscr{E}(p) = E$ (periodically extended in p space) by the plane $p_z = \text{const}$; the orbits which limit these sections are assumed to be closed and not self-intersecting. To calculate the value of i_z, we can utilize the representation of the electron motion in a trajectory determined by integration of its velocity $v = \nabla_p \mathscr{E}(p)$ with respect to time in accordance with the classical law of motion in an orbit in p space[7]:

$$\frac{d\mathbf{p}}{dt} = -\frac{e}{c}[\mathbf{vH}], \text{ i.e. } \frac{dl}{dt} = \frac{e}{c}VH \qquad (3)^*$$

(V is the projection of the vector v on the plane xy, dl is the element of length of the orbit). Here two different cases can be presented.

In an orbit corresponding to given values of n, p_z, if the velocity projection v_z does not change sign (all

[1] We do not take into account the finiteness of the time between collisions of the electrons, which leads to a smearing of the quantized energy levels and to appearance of the attenuating Dingle factor [4] in the expressions for the oscillations.

[2] It is assumed that for fixed p_z, neighboring closed orbits do not approach each other very closely. If this were not so the energy would depend on p_x [5].

*$[\mathbf{vH}] \equiv \mathbf{v} \times \mathbf{H}$.

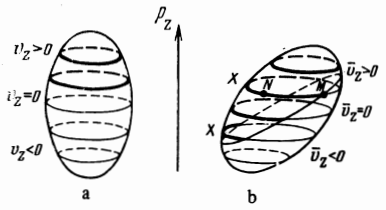

FIG. 1

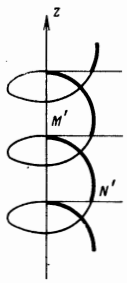

FIG. 2

FIG. 1. Electron orbits in p space. The heavy lines show the portions with $v_z > 0$. a—in all orbits the sign of v_z is constant, b—there are orbits with change of sign of v_z. The crosses indicate a pair of symmetric orbits. The inclined line is the geometric locus of points at which $v_z = 0$.

FIG. 2. Projection of an electron projectory on a plane passing through the z axis (the case when v_z changes sign in an orbit in p space). The heavy lines show the portions in which the electron can leave the metal after executing several turns in the metal.

orbits in Fig. 1a are of this type, and also the upper and lower orbits in Fig. 1b), then the electron motion along the z axis occurs monotonically, similar to the motion of a free electron along a cylindrical screw line. In this case for $v_z > 0$ the electron can leave the metal at any point of its trajectory, so that i_z is the average of the quantity $v_z D$ (D is the transmission coefficient through the barrier in the absence of a magnetic field) over the entire orbit:

$$i_z = \overline{v_z D} = \oint v_z D \frac{dl}{V} \Big/ \oint \frac{dl}{V} \quad (v_z \geqslant 0). \tag{4}$$

For $v_z < 0$ the electron moves in the interior of the metal and, of course, cannot go outside, so that for such states $i_z = 0$.

If v_z changes sign in the orbit (see Fig. 1b, the three middle orbits), then in parts of each turn of the trajectory the electron moves in the positive z direction, and in parts in the negative z direction (see Fig. 2). Consequently, electrons can leave the metal both with $\bar{v}_z > 0$ and with $\bar{v}_z < 0$, where, of course,

$$\bar{v}_z = \oint v_z \frac{dl}{V} \Big/ \oint \frac{dl}{V}. \tag{5}$$

However, a major difference exists between electrons with $\bar{v}_z > 0$ and $\bar{v}_z < 0$. The first type, on arriving from the interior of the metal, describe many turns before their collision with the surface, as a result of which their energy levels are quantized; these electrons contribute both to the monotonic and oscillating components of the current. The second type approach the surface without having completed even one complete turn after the previous reflection from the surface, their levels are not quantized, and they contribute only to the monotonic part of j_z. As a consequence of this, in calculation of the monotonic and oscillating components of the current (they are separated by means of the Poisson summation formula), it is necessary in practice to use different expressions for i_z. The flux density which determines the monotonic component is different from zero for all orbits which have portions with $v_z > 0$ (in Fig. 1 these portions are shown by heavy lines). In view of the central symmetry of the surfaces we can combine the contributions to j_z from symmetric orbits (in Fig. 1b a pair of such orbits is denoted by crosses), after which we must set

$$i_z^{\text{mon}} = \overline{|v_z| D} = \oint |v_z| D \frac{dl}{V} \Big/ \oint \frac{dl}{V} \tag{6}$$

(here D is understood to mean $D(\mathbf{p} \cdot \operatorname{sign} v_z)$), carrying out the summation in (1) only over states with $\bar{v}_z > 0$.

To determine the oscillating part of the current, in Eq. (1) from the very beginning we must take only terms corresponding to states with $\bar{v}_z > 0$. In calculation of i_z it is necessary to take into account that an electron which has first executed a number of turns in the metal can leave it only in a definite part M'N' of each turn of the trajectory (in Fig. 2 these parts are shown by the heavy lines). This gives

$$i_z^{\text{osc}} = \int_{MN} v_z D \frac{dl}{V} \Big/ \oint \frac{dl}{V} (\bar{v}_z \geqslant 0), \tag{7}$$

where MN is the part of the orbit corresponding to the part M'N' of the turn (see Fig. 1b) and defined by the expression

$$\int_{MN} v_z \frac{dl}{V} = \bar{v}_z \oint \frac{dl}{V} = \oint v_z \frac{dl}{V} \quad (\bar{v}_z \geqslant 0). \tag{8}$$

If v_z is everywhere positive in the obit, then M'N' is drawn out to a complete turn, and correspondingly MN spreads out to the entire orbit, and Eqs. (6) and (7) go over to Eq. (4); consequently, there is no further need to make special calculations for the case in which v_z has a constant sign in the orbit.

By using the fact that $\mathbf{v} = \nabla_{\mathbf{p}} \mathcal{E}(\mathbf{p})$, we reduce Eqs. (5) and (6) by simple geometrical constructions to the form

$$\bar{v}_z = -\frac{\partial S}{\partial p_z} \Big/ \frac{\partial S}{\partial E} = E_n'(p_z), \tag{9}$$

$$i_z^{\text{mon}} = \frac{1}{\delta p_z} \int_{\delta\Sigma(E)} D(E, \mathbf{P}) d^2\mathbf{P} \Big/ \frac{\partial S}{\partial E}, \tag{10}$$

where $\delta\Sigma(E)$ is the projection on the plane $p_x p_y$ of the part of the constant-energy surface lying between the plane of the orbit and the plane removed from it by δp_z (see Fig. 3), and $\mathbf{P}$ is the projection of the quasimomentum on the $p_x p_y$ plane. Similarly, Eq. (7) reduces to a form differing from Eq. (10) only in that the integration is carried out not over the entire region of $\delta\Sigma(E)$, but only over the part belonging to the portion MN of the orbit.

Converting in Eq. (1) to the variable of integration $E = E_n(p_z)$ and utilizing Eq. (9), we obtain

$$j_z = -\frac{e^2 H}{h^2 c} \sum_{\sigma} \int dE f(E + \sigma\mu_0 H) \sum_{\substack{n \\ (\bar{v}_z > 0)}} \left(\frac{i_z}{\bar{v}_z}\right)_{nE}. \tag{11}$$

Use of the Poisson summation formula gives

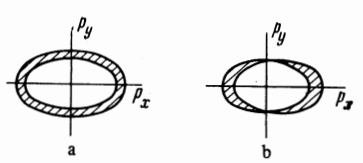

FIG. 3. The region of $\delta\Sigma(E)$ (crosshatched).

$$\sum_{\substack{n\\(\bar v_z>0)}}\left(\frac{i_z}{\bar v_z}\right)_{nE}=\Sigma\left\{\int_{n_{min}(E)}^{n_{max}(E)}\left(\frac{i_z^{non}}{\bar v_z}\right)_{nE}dn+2\sum_{l=1}^{\infty}\mathrm{Re}\int_{n_{min}(E)}^{n_{max}(E)}\left(\frac{i_z^{osc}}{\bar v_z}\right)_{nE}e^{2\pi lni}dn\right\}, \tag{12}$$

where $n_{min}(E)$ and $n_{max}(E)$ are extremal values of the function $n(E, p_z)$ of Eq. (2) for a fixed value of E, and the symbol Σ denotes summation over all intervals in which this function is monotonic, for which $\bar v_z > 0$. Substituting (12) into (11), we obtain

$$j_z=j_z^{mon}+j_z^{osc},$$

$$j_z^{mon}=-\frac{e^2H}{h^2c}\sum_{\sigma}\int dEf(E+\sigma\mu_0H)\sum\int_{n_{min}(E)}^{n_{max}(E)}\left(\frac{i_z^{mon}}{\bar v_z}\right)_{nE}dn, \tag{13}$$

and j_Z^{osc} is determined similarly. We note that the designations "monotonic" and "oscillating" are to some degree arbitrary: j_Z^{mon} contains oscillations due to the oscillating dependence of the chemical potential on H, and j_Z^{osc} contains a small term which is proportional to H^2.

Calculation of the inner integral in Eq. (13) by means of Eqs. (10), (9), and (2) leads to the result

$$j_z^{mon}=-\frac{e}{h^3}\sum_{\sigma}\int\Phi(E)f(E+\sigma\mu_0H)dE, \tag{14}$$

where

$$\Phi(E)=\int_{\Sigma(E)}D(E,\mathbf{P})d^2\mathbf{P},$$

$\Sigma(E)$ is the part of the projection of the surface $\mathcal{E}(p) = E$ on the plane p_xp_y lying in the central Brillouin zone corresponding to a plane lattice. Eq. (14) differs from Eq. (11) of ref. 2 only in the inclusion of spin paramagnetism (not considering the implicit dependence on H through the chemical potential). This is quite natural since Eq. (14) is the result of replacement of the summation over discrete values of n in Eq. (11) by integration, i.e., does not take into account the effect of quantization in the magnetic field.

In order to determine the oscillating component of the current, we will calculate the second term of expression (12). Integrating by parts and keeping only the integrated term, we have

$$\sum\mathrm{Re}\int_{n_{min}(E)}^{n_{max}(E)}\left(\frac{i_z^{osc}}{\bar v_z}\right)_{nE}e^{2\pi lni}dn=\frac{1}{2\pi il}\sum_m[\pm D_m(E)\sin 2\pi l n_m(E)], \tag{15}$$

where $D_m(E) = (i_Z^{osc}/\bar v_Z)_{n_m(E),E}$; here and subsequently $n_m(E)$ is either $n_{max}(E)$ or $n_{min}(E)$, the upper sign referring to the first case and the lower to the second; the symbol $\sum_m$ indicates summation over all extremal sections.

According to Eqs. (9) and (12), in an extremal orbit $\bar v_Z = 0$. Here two cases are possible; a) $v_Z \equiv 0$ (Fig. 1a); b) v_Z changes sign (Fig. 1b).

In case a) the portion MN includes the entire orbit, and from Eqs. (7) and (5) we have

$$D_m(E)=\frac{1}{|\delta S|_{\delta\varepsilon_m(E)}}\int D(E,\mathbf{P})d^2\mathbf{P} \tag{16}$$

(compare with Eq. (10)); here $|\delta S|$ = the areas $\delta\Sigma$, i.e., the effective transmission coefficient in an extremal orbit $D_m(E)$ is the average value of $D(E, \mathbf{P})$ over the region $\delta\Sigma_m(E)$ belonging to this orbit. The transmission coefficient in the absence of the magnetic field, $D(E, \mathbf{P})$, is a rapidly rising function of the quantity $E^{(Z)} = E - P^2/2m_0$:

$$D(E,\mathbf{P})\approx e^{-\xi(E^{(z)})},\ \xi(E^{(z)})=\frac{4}{3}\frac{(2m_0)^{1/2}(-E^{(z)})^{3/2}}{e\hbar F}\Theta\left(\frac{e^{3/2}F^{1/2}}{-E^{(z)}}\right) \tag{17}$$

(see Eqs. (12) and (7) in ref. 2); here m_0 is the free electron mass, F is the electric field applied to the metal, and Θ is the Nordheim function ($\Theta(0) = 1$, $\Theta(1) = 0$[2]); the energy of a fixed electron is taken as zero outside the metal in the absence of a field; it is assumed that $\xi(E^{(Z)}) \gg 1$. Consequently:

$$D_m(E)\sim D(E,\mathbf{P}_m(E))\approx\exp\{-\xi(E-\Delta_m(E))\}, \tag{18}$$

where P_m is the minimal value of P in the m-th extremal orbit, and $\Delta_m = P_m^2/2m_0$.

In case b) the portion MN, according to Eq. (8), is drawn out to the point M, and the effective transmission coefficient, as can be seen from Eqs. (7) and (8), reduces to the value of $D(E, \mathbf{P})$ at this point:

$$D_m(E)=D(E,\mathbf{P}_{Mm}(E))\approx\exp\{-\xi(E-\Delta_m(E))\},\ \Delta_m(E)=P_{Mm}^2(E)/2m_0. \tag{19}$$

We note that in this case the terms discarded in derivation of Eq. (15) may reach, for $H \sim 10^4$ G, the same order of magnitude as the term retained (if the emission occurs from a large group). In this connection we recall that the transmission coefficient $D(E, \mathbf{P})$ given by Eq. (17) is itself determined only with an accuracy to the pre-exponential factor of the order of unity.

It is easy to show that in practice in formula (15) we must use expression (16) for $D_m(E)$ when $\varphi \ll n_m^{-1/2}(E)$ and expression (19) when $\varphi \gg n_m^{-1/2}(E)$ (φ is the angle between the extremal orbit and the curve $v_Z = 0$ at the point of their intersection, i.e., at the point M; in case a), $\varphi = 0$).

2. INVESTIGATION OF OSCILLATING TERMS

On the basis of Eqs. (11)–(13) and (15) we have

$$j_z^{osc}=\frac{e^2H}{\pi h^2c}\sum_{\sigma lm}\frac{\mp1}{l^2}\int D_m(E)f(E+\sigma\mu_0H)\sin 2\pi l n_m(E)dE. \tag{20}$$

We will integrate by parts and discard the nonoscillating integrated term:

$$j_z^{osc}=\frac{e^2H}{2\pi^3h^2c}\sum_{\sigma lm}\frac{\mp1}{l^2}\int\frac{d}{dE}\left[\frac{D_m(E)f(E+\sigma\mu_0H)}{n_m'(E)}\right]\cos 2\pi l n_m(E)dE.$$

At low temperatures the main contribution to this integral is provided by the term with a δ-shaped derivative of the Fermi function. Using the equation

$$\int_{-\infty}^{\infty}f'(E)\cos(qE+\alpha)dE=\Psi(\pi qT)\cos(-qw+\alpha)$$

($-w$ is the chemical potential, w is the work function, $\Psi(u) = u/\mathrm{sh}\,u$, and the temperature is expressed in ergs), we obtain for this contribution

$$j_{z1}^{osc}=\frac{2e}{\pi h^3}\sum(\pm m)(\mu H)^2D_m(-w)\times\sum_{l=1}^{\infty}\frac{(-1)^l}{l^2}\Psi(l\lambda)\cos\left(\pi l\frac{m}{m_0}\right)\cos\left(l\frac{cS_m'(-w)}{e\hbar H}\right); \tag{21}$$

here and subsequently the symbol Σ denotes summation over all extremal sections, $S_m(-w)$ is the area of an extremal section of the Fermi surface,

$$m = S_m'(-w)/2\pi,\ \mu = e\hbar/|m|c,\ \lambda = 2\pi^2 T/\mu H;$$

the chemical potential can be taken the same as in the absence of a magnetic field. On substitution into (21) of expression (16) for D_m, we obtain Eq. (13) from ref. 1, and on substitution of expression (19) we obtain a formula which replaces Eq. (14) in ref. 1.

On comparing Eq. (21) with Eq. (19) and the formulas following from ref. 2, we see that at sufficiently low temperatures ($\Psi(\lambda) \sim 1$) the relatively small size of the oscillations of this type is determined mainly by two parameters: $\mu H/\epsilon(-w)$ (or $\mu H/\zeta$ for $\zeta < \epsilon(-w)$) and $D_m(-w)/D_{max}$. Here $\epsilon(E) = -[\xi(E - \Delta(E))]^{-1}$ (for the usual experimental conditions $\epsilon(-w) \sim 10^{-2}$–$10^{-1}$ eV); ζ is the chemical potential computed from the bottom (top) of the group, $\zeta = |-w - E_g|$; $D_{max} \approx \exp\{-\xi(-W)\}$ is the maximal transmission coefficient for electrons in the metal at T = 0; W is the effective work function for field emission, which is defined by the relation

$$-W = \max_{E \le -w}[E - \Delta(E)],\ \Delta(E) = P_{min}^2(E)/2m_0;$$

$P_{min}(E)$ is the minimal value of P in the region of $\Sigma(E)$ (for more detailed information about these quantities see ref. 2, Eqs. (13), (19), (20), and pages 1430, 1432).

The first of these small parameters in fields $H \sim 10^4$ G for the usual effective electron mass has an order of magnitude 10^{-2}–10^{-3}, and for groups with a small effective mass has an order of magnitude approaching unity. However, the decisive role is played by the parameter $D_m(-w)/D_{max}$, since most of the time it is exponentially small; this is due to the fact that the difference between the arguments of the function ξ and the exponents of D_{max} and $D_m(-w)$, i.e., between $-W$ and $-w - \Delta_m(-w)$ (see Eqs. (18) and (19)), generally speaking, is large in comparison with the small quantity ϵ. The only exceptions are cases in which the maximal value of the transmission coefficient for T = 0 (i.e., in practice the quantity $E - P^2/2m_0$) is achieved near an extremal orbit of the Fermi surface (case a)) or near the point M of this orbit (case b)). For this it is required, first of all, that the quantity $E - \Delta(E)$ have a maximum at the Fermi energy (case 1 of ref. 2, page 1433) or very close to it. Then a number of variants are possible, for example: the groups responsible for the emission are small (for this it is necessary that the larger groups, if they exist, be located in projection on the plane $p_x p_y$ farther from the origin than these small groups); the groups responsible for the emission are not small but are "fortunately" located relative to the p_z axis (for example, as shown in Fig. 1a; obviously, in this case, $P_m = P_{min}$ and $D_m(-w) = D_{max}$).

Equation (21) describes ordinary oscillations of field emission current of the de Haas-van Alphen type, which are due to the alternating passage of the positive and negative half periods of the sine function in the integral of Eq. (20), as H varies, through the point of falloff of the Fermi function (the lower the temperature, the sharper the fall). However, since the transmission coefficient $D_m(E)$, which occurs under the same integral sign, also depends extremely rapidly on the energy, when its maximum occurs below the Fermi level, still another type of oscillation can appear, which is due to passage of the sine-wave half period through this maximum. An interesting feature of these oscillations is their temperature independence[3]. The condition for appreciable value of their amplitude is that the width of the maximum ΔE should not be large in comparison with the period of the sine function, which is $\mu_p H/l$ (the index p indicates that the value is taken for $E = E_p$, where the maximum of $D_m(E)$ occurs).

The function $D_m(E)$ reaches its maximum at the same time as the quantity $E - \Delta_m(E)$, which can occur, for example, for lateral (with respect to the p_z axis) hole groups (compare with ref. 2, page 1433). Near the maximum

$$D_m(E) \approx D_m(E_p) \exp\{-(E - E_p)^2/\Delta E^2\}, \qquad (22)$$

where

$$\Delta E^2 = 2\varepsilon_m(E_p)/\Delta_m''(E_p),\ \varepsilon_m(E) = -[\xi'(E - \Delta_m(E))]^{-1}.$$

If this function extends sufficiently far in both directions from E_p and $-w - E_p \gg \Delta E$ (for a filled group $E_g - E_p \gg \Delta E$), then calculation of the integral in Eq. (20) by means of the formula

$$\int_{-\infty}^{\infty} e^{-q^2x^2}\sin(px + \alpha)\,dx = \frac{\sqrt{\pi}}{q} e^{-p^2/4q^2}\sin\alpha$$

gives

$$j_{z2}^{osc} = \frac{2e^2H}{\sqrt{\pi}hc}\sum(\mp\Delta E)D_m(E_p)$$

$$\sum_{l=1}^{\infty}\frac{(-1)^l}{l} e^{-(\pi l\Delta E/\mu_p H)^2}\sin\left(\frac{cS_m(E_p)}{e\hbar H}\right) \qquad (23)$$

We will now compare the oscillation amplitudes of Eqs. (23) and (21), assuming that the function (22) exists up to $E = -w$. For the condition

$$\mu_p H(-w - E_p)/\pi l\Delta E^2 < 1, \qquad (24)$$

which obviously is usually satisfied, the amplitude of the l-th harmonic of the expression j_{Z1}^{osc} for T = 0 is larger, but with increasing temperature its ratio to the corresponding amplitude j_{Z2}^{osc} rapidly falls. We note further that j_{Z2}^{osc} exists also for filled bands and groups, for which the ordinary oscillations are generally absent. In this case if the maximum value of the transmission coefficient for T = 0 is reached near the energy E_p, i.e., $D_m(E_p) \sim D_{max}$, then oscillations of this type can turn out to be larger than the ordinary oscillations.

For a convenient representation of the order of magnitude of the parameters introduced, we will give specific values for hole groups having the shape of ellipsoids of revolution with an axis parallel to the p_z axis (case a)). In this case $\Delta_m(E) = \Delta(E)$,

$$E_p = E_m = E_g - \frac{|m|\Delta(E_g)}{m_0(1 + |m|/m_0)^3}$$

[3]On inclusion of the Dingle factor (see note 1), of course, a dependence on temperature appears through the time between electron collisions.

(see ref. 2, page 1434),

$$\Delta E^2 = 4\frac{|m|}{m_0}\varepsilon(E_m)\frac{\Delta(E_\varepsilon)}{(1+|m|/m_0)^3}$$

and for condition (24) to be satisfied it is sufficient that

$$\left(1+\frac{|m|}{m_0}\right)\mu_p H/4\pi l\varepsilon(E_m) < 1.$$

Oscillations of a third type, as we have already mentioned, are contained in Eq. (14) and are due to the oscillating dependence of the chemical potential. These oscillations have been discussed exhaustively in an earlier article[1] (it is necessary only to replace eFa_0 by $\epsilon(-w)$). This type of oscillation exists, of course, in all physical quantities but, since they have an order of magnitude ζ_{osc}/ζ, as a rule they are small in comparison with ordinary oscillations. The only exceptions are quantities which depend strongly on the chemical potential: the field emission current, contact potential difference,[8] and the tunnel current between two metals separated by a thin layer of dielectric.[9]

In regard to the experimental observation of field emission current oscillations, the situation is not completely clear. In the abstracts of the X International Conference on Low Temperature Physics (Moscow, 1966), there is a communication by F. J. Blatt (abstract M-77) on observation of these oscillations in bismuth at 4.2°K in fields of 0–50 kG; however, this paper was not delivered. On the other hand, the paper of L. Groman (M-88) at the same conference reports no oscillations in fields up to 40 kG.

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165