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GENERAL FEATURES AND MASTER EQUATIONS FOR STRUCTURIZATION IN COMPLEX DUSTY PLASMAS

V. N. Tsytovich [a]*, ***G. E. Morfill*** [b]**

[a] *General Physics Institute, Russian Academy of Science*
119991, Moscow, Russia

[b] *Max Planck Institute for Extraterrestrial Physics*
G-85740, Garching, Germany

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Dust structurization is considered to be typical for complex plasmas. Homogeneous dusty plasmas are shown to be universally unstable. The dusty plasma structurization instability is similar to the gravitational instability and can results in creation of different compact dust structures. A general approach for investigation of the nonlinear stage of structurization in dusty plasmas is proposed and master equations for the description of self-organized structures are formulated in the general form that can be used for any nonlinear model of dust screening. New effects due to the scattering of ions on the nonlinearly screened grains are calculated: nonlinear ion dust drag force and nonlinear ion diffusion. The physics of confinement of dust and plasma components in the equilibria of compact dust structures is presented and is supported by numerical calculations of master equations. The necessary conditions for the existence of equilibrium structures are found for an arbitrary nonlinearity in dust screening. Features of compact dust structures observed in recent experiments agree with the numerically calculated ones. Some proposals for future experiments in spherical chamber are given.

1. INTRODUCTION

Complex-dusty plasmas have been investigated intensively for about 15 years (see reviews [1–7]). The term “complex plasmas” is used in the case where dust grains have large charges and interact strongly. This last effect is of special interest for plasma crystals discovered in 1964 [8–10], for physics of dust clouds levitating above etching samples in the plasma etching processes [11], and for dust in fusion devices [12–14].

We here consider the systems containing many grains with large charges (the exact meaning of the term “large dust charge” is given somewhat later). For large dust charges, a new physics related to plasma fluxes on dust grains is important [5]. We briefly explain the main new features in complex-dusty plasmas in the case where the dust charges are large (in current experiments, the dust charge in the units of electron charge, Z_d, reaches the value 10^4–10^5 or even larger). The results obtained previously, for example, in the approaches in [4, 15], using investigations of a single dust grain in plasmas, often cannot be transferred to a collection of many dust grains with large charges. The dust charges in plasmas are usually created by plasma fluxes, which increase with an increase in the grain charge. Stationary grain charges should be supported by the presence of continuous electron and ion fluxes with electron–ion recombination inside or on the surface of the grains. In stationary conditions, the process of plasma absorption on grains should be compensated by plasma production by ionization processes. The ionization process can either exist far from grains or be homogeneous in space. In the first case (due to the continuity of fluxes), the flux absorbed on some grain should be present far from this grain at distances about $\lambda_{scr}/a \gg 1$ times larger than the grain charge screening length λ_{scr} (a is the grain size). In most laboratory conditions, many grains are present inside such large radius and obey the drag forces produced by plasma fluxes of different grains within this radius. Thus, the grain interactions are extended to distances much longer than the screening distance and are determined by fluxes produced by many grains. For an enhanced concentration of grains in some space region

*E-mail: tsytov@lpi.ru
**E-mail: gem@mpe.mpg.de

(grain collection), the total flux should have the averaged component directed toward the grain collection and by its ram pressure can increase the grain density enhancement. Once the grain density enhancement appears, it starts to grow and can form a regular dust structure on the nonlinear stage. For homogeneous ionization, the homogeneous dusty plasma can in principle exist initially if the ionization is compensated by plasma absorption on grains. But already in [16] it was shown that such homogeneous distribution is unstable with respect to structure formation. This was found theoretically in the framework of a linear approach. Simultaneously, structures of a different kind were observed in the first experiments on board the International Space Station (ISS) in [17]: dust compact structures, dust voids, and dust vortices were observed. Formation of such structures cannot be determined only by geometry of discharges, by the external voltage configuration or by walls, and so on, and was considered an important intrinsic property of dusty plasmas. Also in first observations of dust clouds in etching experiments [11], a complicated vertical dust distribution in the dust cloud above the sample was observed. It cannot be explained by configuration of the etched sample and must be created by self-organized processes in the dust cloud above the sample. Recently, the laboratory experiments in [18] showed the formation of complicated dust structures with dust density peaks. This was found in new regimes of devices that have been previously used for investigation of dust acoustic waves [19, 20].

The linear dust structurization instability has many features in common with the known gravitational instability. The growth rate for both of them is nonvanishing for zero wave numbers. In this sense, both instabilities are universal. The scales are quite different, however. We recall that the gravitational instability leads to an important phenomenon, gravitational structurization of matter. The dust structurization instability can be expected to have similar consequences in complex-dusty plasmas. Until present, no theoretical efforts have been made to trace how the linear complex-dusty plasma structurization instability converts to its nonlinear stage and whether some stable dust structures can be finally formed. The main problem is currently to describe and scan the possible structures in their nonlinear stationary stage because the time required for structure formation in most existing experiments is rather short and the observations can provide only the final stage of structurization, the dust distributions in stationary dust structures. The theoretical problem is to find the equilibria of nonlinear structures and to scan possible types of equilibrium dust structures. Here, we explain the physics of equilibrium that can be established in stationary nonlinear dust structures and present their general mathematical description by formulating of a set of nonlinear master equations. The theoretical predictions can have applications for three most important issues in complex-dusty plasmas: plasma crystal formation [8–10], plasma etching processes [11], and dust in modern fusion devices [12–14].

Several processes are important for the nonlinear structure formation with large grain charges. The nonlinearity in grain charge screening determines the drag force for dust sizes larger than $(0.1–0.2)\mu$m. This is the case for typical plasma crystal experiments, where the grain size is about 10μm. The nonlinearity in grain charge screening was partly investigated in [21–23] only at a relatively small distance from the grain, although the role of the nonlinearly at larger distances may not be negligibly small. The diffusion due to ion scattering on nonlinearly screened highly charged grains have been completely neglected so far.

Also some simplifying assumptions have been used in previous investigations, such as quasineutrality [24], which can be violated inside the structures in presence of large drag forces. Different types of nonlinearities in screening could be important [25] and are still waiting to be investigated. It is therefore desirable to formulate the problems for an arbitrary nonlinearity in dust charge screening and investigate the influence of the nonlinearity on both the drag force and the diffusion coefficient and formulate the master equations taking the nonlinearity into account in both the ion drag process and the diffusion process. This is one of the aims of this paper.

We show here that some general results for the range of existence of structure equilibria and their properties can be obtained independently of the model of the nonlinearity. We exclude the gravity forces from consideration, having in mind possible applications to microgravity experiments. The forces maintaining the grains and plasma components in a compact region in space include the plasma collective fluxes directed toward the structures due to absorption of the plasma fluxes on grains. In the distant past, the model of gravity as shadowing of the ether flux absorption was proposed by Le Sage (see [4]). The real fluxes of electrons and ions on the grains in dusty plasmas have an effect similar to the Le Sage gravity, but they are produced collectively by the whole dust density enhancement.

From a general theoretical standpoint, the plasma fluxes in the structure can have regular and random parts. The latter change the grain interaction as discussed in [3]. In this paper, we deal only with the regular part of the fluxes using an average hydrodynamic-type approach, i. e., the balance continuity equation in which the binary grain interactions cancel. For simplicity, we consider structures of spherical shape, assuming that they are in a stationary equilibrium. We describe the possible range of parameters that determine the existence of the equilibrium. After a discussion of some general structure features, we derive the master equation that can describe distributions inside the equilibrium dust structures for any model of nonlinear dust charge screening. Next, we consider a special model of nonlinearity in screening [21, 22] and use exact solutions for dust charge screening in this model. With this type of nonlinear screening, we find a new description of the ion dust drag force and ion diffusion, both precesses being determined by ion collisions with nonlinearly screened dust charges. A new effect formulated in this paper is a suppression of diffusion in regions of high dust density that are formed in dust structurization process. The role of dust in the diffusion process was previously not considered for either linear or nonlinear dust screening, and is investigated here for the first time. We present some examples of numerical solutions of the master equations for spherical structures using numerically calculated drag and diffusion coefficients. We compare the results with recent experiments [18, 26] and conclude with discussions and recommendations for future experiments in spherical plasma chambers.

2. PHYSICS AND GENERAL FEATURES OF A DUST STATIONARY COMPACT STRUCTURE WITH HIGHLY CHARGED GRAINS

Some general structure features can be expected from physics of plasma fluxes in dusty plasmas. We give preliminary hand-waving arguments for general features of dust structures, which are in agreement with present investigations of the exact solutions of the master equations. We consider the limit of highly charged dust grains $Z_d = |Q_d/e| \gg 1$, which is rather important for structure formation in typical modern experiments. The large charges can cause not only strong grain interactions. Among new effects are the nonlinear screening of grain fields, excitation of regular electric fields, changes of dust ion drag forces, and dust suppression of ion diffusion. We start with a qualitative description of possible equilibria of dust structures.

2.1. Regular electric fields generated in dust structures by a plasma flux

We consider systems with sizes much larger than the nonlinear screening length of an individual grain. We explain why the presence of an averaged collective field is inevitable outside the screening length. The main reason is the presence of regular plasma fluxes producing dust drag forces F_{dr} that are usually proportional to the plasma flux nu, $F_{dr} \propto nu$ (u is the directed flux velocity and n is the ion density). For highly charged grains, F_{dr} is proportional to $Z_d \gg 1$. We clarify this point. For linear screening, the drag force is proportional to Z_d^2 since the cross section of scattering of ions on grains is usually estimated as πr_{eff}^2 with $r_{eff} = Z_d e^2/T_i$. But one factor Z_d enters the condition for the screening to be linear $Z_d e^2/\lambda_D T_i \equiv \beta \ll 1$, where λ_D is the ion screening length, and $\lambda_D \propto 1/\sqrt{n}$ for $\tau = T_i/T_e \ll 1$. This shows that $F_{dr} \propto Z_d \beta \sqrt{n} u$. With an increase in Z_d, the nonlinearity in screening starts to be important for $\beta > 1$, changing the dependence of F_{dr} on β. It is more useful to define F_{dr} with one factor Z_d and write $F_{dr} = Z_d f_{dr} \sqrt{n} u$ with the coefficient f_{dr} depending on both β and $|u|$ (it in fact increases with β for $\beta \ll 1$ and decreases with β for $\beta \gg 1$). We say that gains are large charged grains if $\beta > 1$.

In the balance of forces on grains, apart from the drag force, the electric field force $F_{el} \propto Z_d E$ should be first of all taken into account (E being the collective electric field strength), since this force also contains a large factor Z_d for highly charged grains. The balance of the drag force and the electric field force results in some kind of Ohm's "law" $u \propto E$, showing that the ion drift velocity is proportional to the electric field strength (with a coefficient depending on $|u|$ and β). This law can be read in both directions: the electric field excites the ion drift or the ion drag force excites the electric field. The second option is important for structure investigation because the ion flux is inevitably created by grain charging. The other forces acting on grains cannot contribute much to the balance of forces comparable with the contribution of drag and electric field forces even for strongly interacting grains. The main point is that the drag force excites regular electric fields, which can often be present in the complex plasma structures. Although the collective field strength can be moderate, the force that it produces on a grain with $Z_d \gg 1$ is large. An interpretation of the recent observations in [26] gives evidence that such collective fields are indeed exited in the structures.

2.2. Physics of dust and plasma confinement in a single structure

We consider an example of a single spherical structure. Inside the structures, the dust and ion density are enhanced. In the absence of dust and for sudden ionization of a spherical volume, the electrons (as the lightest particles) leave the surface and create a polarization field acting on ions, which results in the known "Coulomb explosion". In the presence of dust, the picture is quite different if the drag forces exceed the polarization forces. The drag is directed inwards to the center and creates an electric field that balances it. The field is directed inwards due to negative charges of the dust grains. For ions, this force is an inwards force (not outwards as in the case of Coulomb explosion). This means that the ions have an electrostatic potential well inside the structure. The electrons adiabatically follow the electric field distribution. This effect occurs because of the large dust charges $Z_d \gg 1$ and also because these charges have to be supported by plasma fluxes. This qualitative picture of dust and plasma particle confinement in the structure is supported by numerical calculations of balance equations.

2.3. Normalization

To explicitly formulate the balance, which we have explained qualitatively, we introduce the dimensionless variables that are useful in low-temperature plasma experiments in the case of a high neutral atom density. The neutrals rapidly equalize the ion and neutral atom temperature denoted here by T_i, and the ion temperature can be assumed to be constant inside the structure volume. We introduce the ion mean free path λ for interaction of ions with neutrals and consider it constant in the volume of the structure (for a constant cross section σ_{in} of ion–neutral collisions $\lambda = 1/n_n\sigma_{in}$, with n_n being the neutral density). In modern complex plasma experiments, T_i and λ are often constant. It is therefore useful to normalize all values with respect to quantities containing only these constant values. We use the following normalization of the densities of ions n_i and electrons n_e and the dust charge density $Z_d n_d$ (we call the normalized value of the latter the Havnes parameter P, although an other definition of the Havnes parameter with an other normalization exists in the literature):

$$n \to \frac{n_i}{n_{eff}}, \quad n_e \to \frac{n_e}{n_{eff}}, \quad P \equiv \frac{n_d Z_d}{n_{eff}},$$

where

$$n_{eff} \equiv \frac{T_i}{\lambda^2 4\pi e^2} = 10^7 \text{cm}^{-3} \cdot \left(\frac{n_n}{10^{16}\ \text{cm}^{-3}}\right)^2 \times \times \left(\frac{\sigma_{in}}{3\cdot 10^{-15}\ \text{cm}^{-2}}\right)^2 \frac{T_i}{0.02\ \text{eV}}. \quad (1)$$

We use the following normalization of forces F, electric field strength E, and plasma flux Φ: $F \to F\lambda/T_i$, $E \to \to eE\lambda/T_i$, and $\Phi \to \Phi/\sqrt{2}n_{eff}u$, where $u \to u_i/\sqrt{2}v_{Ti}$ and $v_{Ti} = \sqrt{T_i/m_i}$, while u_i is the actual ion drift velocity and v_{Ti} is the ion thermal velocity. The definition of $f_{dr}(u,\beta)$ follows from

$$F_{dr} \equiv f_{dr}(|u|,\beta) Z_d u\sqrt{n},$$

where β characterizes the ratio of the ion–grain electrostatic energy at the linear screening length $Z_d e^2/\lambda_{scr}$ to the ion average kinetic energy T_i. It can be written as $\beta = Z_d e^2/\lambda_D T_i = za\sqrt{n}/\tau$, where $z = Z_d e^2/aT_e\lambda$ is the normalized dust charge and $\tau = T_i/T_e$ is the ion-to-electron temperature ratio (T_e is also assumed to be constant inside the volume of the structure). We normalize the distances r from the structure center and the grain sizes with respect to the ion–neutral mean free path $r \to r/\lambda$ and $a \to a/\lambda$.

2.4. Balance conditions

With the above notation, the dust balance equation (after cancelation of the common factor Z_d) has the form

$$E = f_{dr}(u,\beta)\sqrt{n}u. \quad (2)$$

The ion balance equation describes the balance of the thermal ion pressure force, the electric field force, and the friction forces on neutrals and electrons:

$$2u\frac{du}{dr} + \frac{1}{n}\frac{dn}{dr} = E - f_{fr,id}u - uf_{fr}.$$

The left-hand side contains the standard expressions of ion hydrodynamics, the normalized term $(\mathbf{v}_i \cdot \nabla)\mathbf{v}_i$ and the ion pressure term $(1/n)(dn/dr)$. The right-hand side contains three terms: $-uf_{fr,id}$ is the ion friction force due to dust drag and $-uf_{fr}$ is the ion friction force due to ion–neutral collisions. The $f_{fr,id}$ term can be found from the ion drag force using the momentum conservation law. Taking expression (2) into account, we have

$$E - f_{fr,id}u = f_{dr}(|u|,\beta)\sqrt{n}(1 - P/n)u.$$

The ion neutral friction force $uf_{fr}(u)$ can be found using the assumption of a constant cross section of ion–neutral collisions in the BGK approach and averaging with respect to the drifting thermal distribution

$$f_{fr}(u) = \sqrt{\left(\frac{8}{3\sqrt{\pi}}\right)^2 + u^2}. \quad (3)$$

The numerical coefficient in the radicand in (3) has been found analytically and the u dependence is found using a best fit with the numerical results. This dependence of the friction on the drift velocity is in accordance with most measurements of ion mobility in low-temperature plasmas [27]. The equation obtained above can serve as a first balance equation for du/dr describing the changes in ion drift velocity. The flux Φ can be divided into the convective flux nu and the diffusion flux $-Ddn/dr$, with D being the diffusion coefficient:

$$\Phi = nu - D\frac{dn}{dr}.$$

The diffusion coefficient should take both the ion–neutral collisions and ion dust collisions into account and is inversely proportional to the sum of frequencies for these collisions. Then the Havnes parameter enters D only in the combination $P/2\sqrt{n}$. The coefficient $D = D(u, \beta, P/2\sqrt{n}\,)$ can be calculated from ion dust cross sections for any model of nonlinear dust screening. Formula (3) can be used as an equation for the dust density derivative dn/dr. The balance of fluxes can be obtained if we account for the ionization (assuming that it is proportional to the electron density n_e) and the rate of absorption of ions on dust:

$$\frac{1}{r^2}\frac{d(r^2\Phi)}{dr} = \alpha_i n_e - anP\alpha_{ch}(u), \tag{4}$$

where α_i is the ionization coefficient depending on the power of ionization and $\alpha_{ch}(u) = 1/2\sqrt{\pi + 4u^2}$ is the charging coefficient obtained from the balance of electron and ion currents on the grain surface. To close the system of balance equations, we use the Poisson equation for the average electric regular electric field in the structure:

$$\frac{1}{r^2}\frac{dEr^2}{dr} = n - n_e - P. \tag{5}$$

According to (2), the value of the field strength E in the left-hand side of (5) depends only on n, u, and z. The derivatives dn/dr and du/dr in (5) can be found from the above equations and are functions of n, u, P, Φ, and r, while the value of dz/dr can be found from the charging equation

$$\exp(-z) = \frac{zn\alpha_{ch}(u)}{n_e\sqrt{m\tau}}, \quad m = \frac{m_i}{m_e},$$

which describes the equality of electron and ion fluxes on the individual grain. Thus all derivatives in the left-hand side of (5) can be found as algebraic functions of n, n_e, u, Φ, r, and P. Equation (5) is then an algebraic equation for P, giving P as a function of n, n_e, u, Φ, and r. Taking into account that both electrons and ions are created in pairs by the ionization source and are also absorbed in pairs by the dust and using the standard electron diffusion and friction on neutrals, we find the balance equation for electrons

$$E = -\frac{\gamma}{\tau}\frac{1}{n_e}\frac{dn_e}{dr}, \quad \gamma = \frac{1}{1 + 9/4\pi},$$

which differs from the simple balance of the electron electric field force and the electron pressure force (where $\gamma = 1$) and is valid in the case where the collective electric fields of the structure substantially exceed the ambi-polar field (the ambi-polar diffusion is thus violated in dust structures).

It is important to explain on a qualitative level why the value $f_{dr}(|u|, \beta)$ in fact also determines the possible equilibria and the degree of quasineutrality in the structures.

2.5. Nonlinear drag force and the range of existence of equilibria of dust structures

To find the general conditions for the existence of equilibrium structures, we use the balance relation at the center of the structure. As $r \to 0$, we have $u \to u_0' r$, $n_e = n_e(0)$, $P = P(0)$, $E \to E_0' r$, $\Phi \to \Phi_0' r$, and $\Phi_0' = n(0)u_0'$, $f_{dr} \to f_{dr,0}(\beta)$, $f_{dr,0}(\beta) = f_{dr}(0, \beta_0)$, $\beta_0 = z_0\sqrt{n(0)}a/\tau$, where $n(0), n_e(0), P(0)$, and $z(0)$ are the normalized ion density, electron density, Havnes parameter, and dust charge at the center of the structure. The equilibrium equation gives the following necessary relations for the existence of equilibrium in the dust structure:

$$\begin{gathered} u_0' = \frac{f_{dr,0}(\beta)}{2\sqrt{n(0)}}(n(0) - P(0)) - \frac{4}{3\sqrt{\pi}}, \\ 3f_{dr,0}(\beta)\sqrt{n(0)}u_0' = n(0) - n_e(0) - P(0), \\ 3u_0' = \alpha_i\frac{n_e(0)}{n(0)} - \frac{1}{2\sqrt{\pi}}aP(0). \end{gathered} \tag{6}$$

These equations can be used for the analysis of possible equilibria independently of the models for nonlinear screening if $f_{dr,0}(\beta)$ is known. The equilibrium is determined only by two parameters, $n(0)$ and the ionization rate α_i, which are related to the two parameters that can be regulated in experiments, the total flux on the surface of the structure and the power of volume ionization. In the simplest case $\alpha_i = 0$, the external flux is a single parameter that determines the structure. The balance can occur only in a restricted range of this parameter determined by (6). There is a one-to-one dependence of the ion density at the center of the structure and the external plasma flux. We can use (6) to express the value of the derivative of the drift flux velocity u_0' at the center, the electron density $n_e(0)$ at the center, and the Havnes parameter $P(0)$ at the center as functions of this single parameter $n(0)$. We find

$$u_0' = -\frac{2}{3\sqrt{\pi}}\frac{n(0)-n_{min}}{n_{max}-n_{min}},$$
$$P(0) = n_{max}\frac{n(0)-n_{min}}{n_{max}-n_{min}}, \quad (7)$$
$$n_e(0) = \frac{n_{max}-n(0)+(3/2)(f_{dr,0})^2(n(0)-n_{min})}{n_{max}/n_{min}-1}.$$

In these relations, the minimum and maximum ion densities are

$$n_{min} = \frac{8\sqrt{n(0)}}{3f_{dr,0}\sqrt{\pi}}, \quad n_{max} = \frac{8}{a}.$$

It follows that for $n_{min} < n(0) < n_{max}$, the Havnes parameter and the electron density are always positive and the flux drift velocity is directed to the center of the structure. We note that the requirement $P(0) > 0$ (the existence of dust at the center) automatically implies that the ion flux close to the center is directed to the center and that $n_e(0) > 0$. The range in which the equilibrium exists is completely determined by the drag force at the center $f_{dr,0}(\beta)$ and the ratio of the grain size to the mean free path a for ion–neutral gas collisions. A generalization of relations (7) to the case where the volume ionization is not zero, $\alpha_i \neq 0$, can be found from (7). It shows that the range of existence of equilibria is narrowed if the ionization rate increases. The conclusion is that the ionization rate should not exceed a critical value, otherwise equilibrium is not possible. The nonlinearity in screening can qualitatively change the dependence of $f_{dr,0}$ on β (for linear screening, i. e., $\beta \ll 1$, the standard expression shows that $f_{dr,0} = \beta\ln(1/\beta) \propto a$, while for nonlinear screening, i. e., $\beta \gg 1$, $f_{dr,0}$ is a decreasing function of a and β in most cases). All these relations are necessary but not sufficient requirements for the existence of equilibrium and they imply the absence of dust void in the center of the structure ($P(0) \neq 0, u_0' < 0$). If they are not satisfied at the center, the balance conditions can start to be satisfied at some distance from the center, which means that a void is present at the center. An important general statement is that in the absence of volume ionization, the equilibrium is determined by a single parameter that can vary only in a limited range. With an increase in the ionization coefficient α_i, this range narrows and only structures with a void in the center can satisfy the conditions for the equilibrium.

2.6. Drag force determines the degree of quasineutrality in the dust structures

In the absence of dust on scales larger than the screening length, the plasma is usually quasineutral. The presence of highly charged grains and their drag by plasma fluxes changes the usual quasineutrality by drag excitation of electric fields. But the approximate quasineutrality, including not only electrons and ions but also the dust particles, can still be present being regulated by the drag force. The quasineutrality means that $n - P - n_e \ll n$ in dusty plasmas. This relation can be checked in the whole structure if we calculate the whole distributions of electron, ions, and grains in the structure. But its validity can be checked using the asymptotic expressions at the center of the structure given above. We find

$$\delta\rho = \frac{n(0)-n_e(0)-P(0)}{n(0)} = -\frac{3}{4}f_{dr,0}^2\frac{1-n_{min}/n(0)}{n_{max}/n_{min}-1}. \quad (8)$$

Since $f_{dr,0} \ll 1$ for the existing models, the system inside the structures is almost neutral but the deviations from quasineutrality are determined by the drag force. For the density $n(0)$ close to its maximum value and for $n_{max} \gg n_{min}$, the electron density at the center is small, which means that in this limit, the negative dust charges at the center are almost compensated by the volume positive ion charges. At the periphery, where the dust density vanishes, the electron and ion densities are close to each other, but are not exactly equal. The electric field can be left at the surface of the structure, which means that the structures can be charged.

2.7. Finite size of the compact dust structures

If the external plasma flux is finite and the volume ionization is absent, the size of the structure should also be finite. This is due to finite value of the Havnes parameter at the center, which cannot stay constant in the structure. The Havnes parameter cannot increase indefinitely or oscillate indefinitely since in that case, the external flux cannot support the dust charges. Therefore, the Havnes parameter should typically vanish at a certain distance, which can be defined as the structure size R_{str}. This means that any equilibrium structure can confine only a finite number of grains. This does not mean that in the presence of ionization outside the structure, the conditions for equilibria can be again satisfied. Thus, being surrounded by a dust void, the structure can have several shells. It can happen that the dust structures inside them have a void and an other void can surround the structure, but the number of grains between the voids in the structure is finite.

2.8. Charges of dust structures and formation of dust clusters

Since the ions dominate in dust drag, we can roughly estimate the total charge of the dust structure to have a potential of the order T_i. Letting E_s denote the dimensionless electric field at the dust structure surface, R_s denote the size (in units of the mean free path of ions in neutral gas), and $-Z_s e$ the total structure charge (the individual grain charge is $-Z_d e$), we find the estimate

$$Z_s = Z_d \frac{a}{z\tau} E_s R_s^2.$$

The numerical models often give $|E_s| > 1$ and $R_s \gg 1$. Therefore, the structures can be highly charged with the total charges exceeding the individual grain charge. In this case, the structures may be regarded as "supergrains" that can form clusters of dust structures. Multiple structure formation is expected for large volumes with a constant volume ionization source, because the condition of the overall balance means that the plasma produced by the source should be absorbed by the structures, while each structure is able to absorb only a finite plasma flux. The possibility of formation of super-crystals in complex plasmas (where an element of the super-crystal is a single dust structure) was mentioned in [27]. This situation is in some sense similar to gravitational cluster formation.

3. MASTER EQUATIONS FOR STRUCTURES

3.1. General form of the master equations

As the master equations, we use the balance equations among which are the equation for ion density balance

$$\textbf{I} \qquad \frac{dn}{dr} = \frac{1}{D\left(u,\beta,P/2\sqrt{n}\right)}(nu-\Phi), \tag{9}$$

the electron density equation

$$\textbf{II} \qquad \frac{dn_e}{dr} = -\frac{\tau}{\gamma} f_{dr}(u,\beta) u n_e \sqrt{n}, \tag{10}$$

the equation for ion drift velocity

$$\textbf{III} \qquad \frac{du}{dr} = \frac{1}{2}\left(f_{dr}(u,\beta)\sqrt{n}u\left(1-\frac{P}{n}\right) - f_{fr}(u) - \frac{1}{nu}\frac{dn}{dr}\right), \tag{11}$$

where dn/dr can be substituted from (9), and the equation for plasma flux

$$\textbf{IV} \qquad \frac{d\Phi}{dr} = -\frac{2\Phi}{r} + \alpha_i n_e - anP\alpha_{ch}(u) \tag{12}$$

obtained from (4). Two more equations are algebraic equations.

1) The equation for the individual dust charge z, explicitly given above in the simplest form and referred to as the master equation **V** in what follows, with its solution denoted by $z_s(u,n,n_e)$. The corresponding drag force is denoted by

$$f_{dr,s}(u,n,n_e) = f_{dr}\left(u, z_s(u,n,n_e)a\sqrt{n}/\tau\right).$$

2) The equation for the Havnes parameter P whose explicit expression can be obtained by differentiating Poisson equation (4):

$$\textbf{Ia} \qquad \frac{dn}{dr} = \frac{nu}{C}(n - n_e - A - PB), \tag{13}$$

or, using (9),

$$\textbf{VI} \qquad D\left(u,\beta,P/2\sqrt{n}\right) u(n-n_e-A+BP) = C(nu-\Phi), \tag{14}$$

where

$$A(u,n,n_e,r) = f_{dr,s}^2(u,n,n_e)\frac{n}{2} - f_{dr,s}(u,n,n_e)F_{fr}(u)\frac{\sqrt{n}}{2} + \frac{2f_{dr,s}(u,n,n_e)\sqrt{n}u}{r} - \frac{nu^2\tau}{\gamma(z_s(u,n,n_e)+1)} f_{dr,s}(u,n,n_e) \times \beta\left.\frac{df_{dr}(u,\beta)}{d\beta}\right|_{\beta=z_s(n,n_e,u)\sqrt{n}a/\tau} + \sqrt{n}S(u,n,n_e,r) \times \left(F_{dr}(u,n,n_e,r)\sqrt{n} - F_{fr}(u)\right), \tag{15}$$

$$B(u,n,n_e) = \frac{f_{dr,s}^2(u,n,n_e)}{2} + f_{dr,s}(u,n,n_e)S(u,n,n_e), \tag{16}$$

$$C(u,n,n_e) = -f_{dr,s}(u,n,n_e)\frac{\sqrt{n}}{2} + f_{dr,s}(u,n,n_e)u^2\frac{\sqrt{n}}{2} + \frac{z_s(u,n,n_e)-1}{2(z_s(u,n,n_e)+1)}\sqrt{n}u^2 \times \beta\left.\frac{df_{dr}(u,\beta)}{d\beta}\right|_{\beta=z_s(n,n_e,u)\sqrt{n}a/\tau} - \sqrt{n}S(u,n,n_e), \tag{17}$$

$$S(u,n,n_e) = \frac{f_{ch}(u)u^2}{z_s(u,n,n_e)+1} \times \\ \times \beta \left. \frac{df_{dr}(u,\beta)}{d\beta} \right|_{\beta = z_s(n,n_e,u)\sqrt{n}a/\tau} + \\ + \frac{u}{2} \left. \frac{df_{dr}(u,\beta)}{du} \right|_{\beta = z_s(n,n_e,u)\sqrt{n}a/\tau}. \quad (18)$$

3.2. Master equations for small diffusion

If the diffusion coefficient is zero, $D = 0$, then $\Phi = nu$ and Eq. (9) cannot be used as a master equation for the ion density because it requires resolving the uncertainty $0/0$, but we can use Eq. (13) instead (also denoted by **Ia** since it is independent of whether diffusion flux is taken into account). As equations **IIa** and **IIIa**, we use (10) and (11), while in the balance of fluxes in (12), we substitute $\Phi = nu$ and both derivatives dn/dr and du/dr taken from **Ia** and **IIIa**. This leads to an analytic expression for the Havnes parameter P:

$$P(u,n,n_e,r) = \frac{M(u,n,n_e,r)}{N(u,n,n_e)}, \quad (19)$$

where

$$M(u,n,n_e,r) = \left(f_{dr,s}(u,n,n_e)\sqrt{n} - F_{fr}(u) + \frac{4u}{r} \right) \times \\ \times C(u,n,n_e) + (2u^2-1)\left(n - n_e - A(u,n,n_e,r)\right), \quad (20)$$

$$N(u,n,n_e) = (1 - B(u,n,n_e))(2u^2-1) + \\ + \left(\frac{f_{dr,s}(u,n,n_e)}{\sqrt{n}} - 2a\alpha_{ch}(u) \right) C(u,n,n_e). \quad (21)$$

The master equations that we have derived are similar to those used for describing self-organization processes [28–30]. The solutions of the master equation can correspond to dissipative self-organized structures.

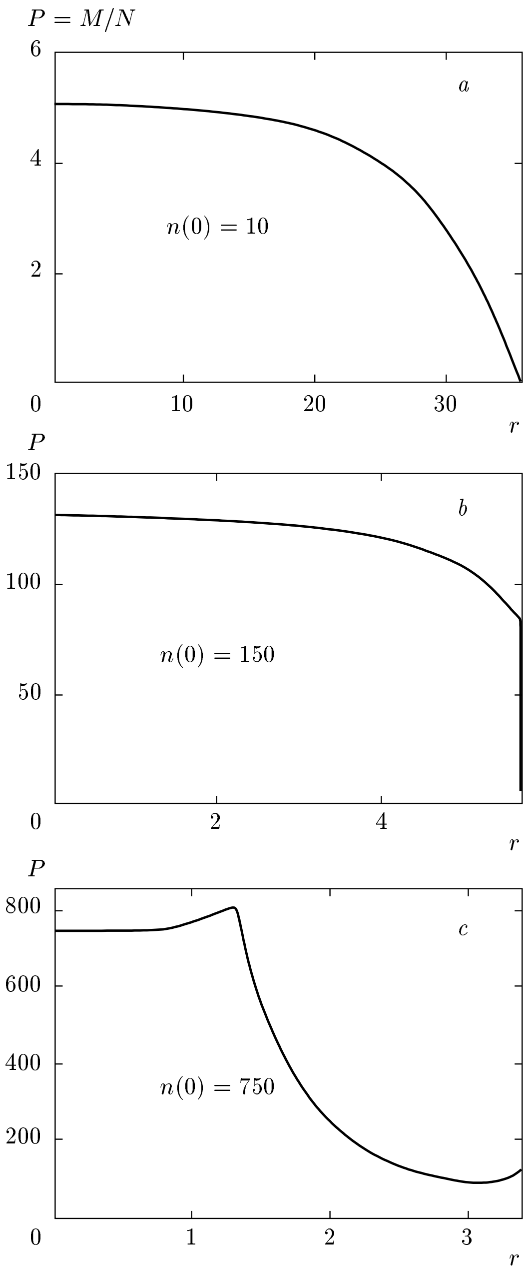

Fig. 1. Dependences of the parameter P in the structures on the distance from their center for three types (1 (a), 2 (b), 3 (c)) of structures, $a = 0.01$, $\tau = 0.01$, gas Argon

4. EXAMPLES OF SOLUTIONS OF THE MASTER EQUATIONS. A NEW TYPE OF DUST SELF-ORGANIZED STRUCTURES

The obtained master equations can be used in numerical investigations of a broad range of different structures, with different nonlinearities in screening, drag forces, and diffusion. They can also be used for some anomalous processes. The equations are also valid for standard linear screening, linear drag, and diffusion on neutrals (they are valid for rather small grains). The nonlinearity can introduce new effects and can be responsible for a new type of dust structures. We here present only several examples of solutions of the master equations with a qualitative description of the main new features obtained in numerical investigation of dust structures. All our numerical results support the general features of structures described above.

Specifically, we present here the results of numerical calculations for the model of nonlinear screening in [21, 22], used to find $f_{dr}(u,\beta)$ and $D(u,\beta,P/2\sqrt{n})$

numerically for $a = 0.01$ and $\tau = 0.01$ (Argon gas) as continuous functions of their arguments that can be used in the master equations. For $\alpha_i = 0$, $z(0)$ was calculated numerically and was used to find the range $2.5 < n(0) < 800$ of the existence of structure equilibrium. This range was divided into 3 subranges: 1) small ion and dust density at the center, $2.5 < n(0) < n_{cr}^{(1)}$, 2) medium ion density at the center, $n_{cr}^{(1)} < n(0) < n_{cr}^{(2)}$, and 3) large ion density at the center, $n_{cr}^{(2)} < n(0) < 800$. The calculations give $n_{cr}^{(2)} \approx 40$ and $n_{cr}^{(2)} \approx 300$. In the first subrange, P has a maximum at the center and decreases slowly to the structure edge $r = R_{str}$, where it vanishes. The lower $n(0)$ is, the larger R_{str}, which can reach the values 30–50; in this subrange, the structure with the largest $n(0)$ has the smallest size about 10. The structure has a finite number of grains confined in it, and this number increases with the structure size. In the second subrange, P smoothly decreases from the center, reaching the distance where it sharply decreases to zero. In the third subrange, the dust density increases from the center, forming one or several peaks and then decreases to zero, also mostly sharply at the structure edge. Figure 1 illustrates the results of numerical computations in the three subranges for $n(0) = 10, 150$, and 750. The calculated structure types can be used to find in which part of the structure the dust crystallization starts and from where the crystallization fronts can propagate. Any crystallization model provides with a criterion of crystallization for the coupling constant $\Gamma_{coupl} > \Gamma_{cr}$, where the coupling constat Γ_{coupl} is proportional to the dimensionless coupling constant $\Gamma \equiv z^{5/3}P^{1/3}$, which is calculated using the obtained results for the structures. The coefficient between Γ_{coupl} and Γ is found to be of the order of 3000 and the crystallization criterion can be fitted with the existing experiments. The dependence of Γ on the distance from the center calculated for first type of structures is shown in Fig. 2 for $n(0) = 10$ (close to n_{min}) and $n(0) = 40$ (close to $n_{cr}^{(1)}$). In the first subrange for the smallest $n(0)$, the crystallization starts at the center, but for larger $n(0) = 40$, the crystallization already starts at distances close to the periphery, although not exactly at the edges of the structure (Fig. 2). For the second type of structures, the crystallization starts almost at the center, but soon covers the whole structure, which corresponds to a rather high speed of the crystallization front. For the third type of structures, the crystallization is complicated due to the presence of dust density peaks, which are the positions where crystallization starts. The calculations have been performed neglecting the diffusion flux, and the latter was calculated by perturbation using an exact expression for diffusion found in the model in [21, 22] by taking both ion–neutral and ion–dust collisions on nonlinearly screened grains into account. It is found that the ion grain collisions strongly suppress the diffusion, especially in the ranges of medium and large dust density structures and that in all subranges, the diffusion fluxes are small compared to the convective fluxes taken into account in first-order calculations (the calculations show that the estimated ratio of diffusion-to-convective fluxes is not larger than (1–2) %). An exception is the narrow region close to the edges of the structure, where due to the absence of diffusion, suppression of the diffusion fluxes by dust can reach the value about 0.5 of the convective fluxes. Therefore, for the structure edges, the present results can give only a rough estimate of the edge shape (a thin layer at the edge), because they are on the limits of the applicability of the approximation used in the calculations. The ratio of the diffusion-to-convection flux calculated for first type of structures is demonstrated in Fig. 2. The structures are charged and the collective electric field at the edge E_s determines the total structure charge Z_s, which is shown in Fig. 2 together with z_s ($z_{str} = Z_s e^2/aT_e$). In Fig. 2, z_s is the individual dust charge at the surface of the structure. For all structures, the electron density n_e and the individual dust charge z increase to the periphery of the structures.

5. DISCUSSION

1. All the obtained master equations are the first-order equations in derivatives and can be solved numerically starting with the asymptotic solutions at the center described here. The difficulties that can appear are due to a new type of "singularities" that can occur inside the structures. Indeed, according to relation (19), the denominator of the expression for the Havnes parameter, $N(u, n, n_e)$, can have zeros in its dependence on the center of the structure. In particular, the peaks shown in Fig. 1 are related to these "singularities". At these points, the values of P are finite but its derivatives change sharply, which indicates that self-consistent solutions always describe the simultaneous zeros of both $N(u, n, n_e)$ and $M(u, n, n_e)$. That is a remarkable feature showing that the master equations do not allow any infinite values of P. Physically, this is reasonable, but the mathematical aspect requires further analyzes.

2. The calculated diffusion flux is found to be small even at peaks of the dust density inside the structures.

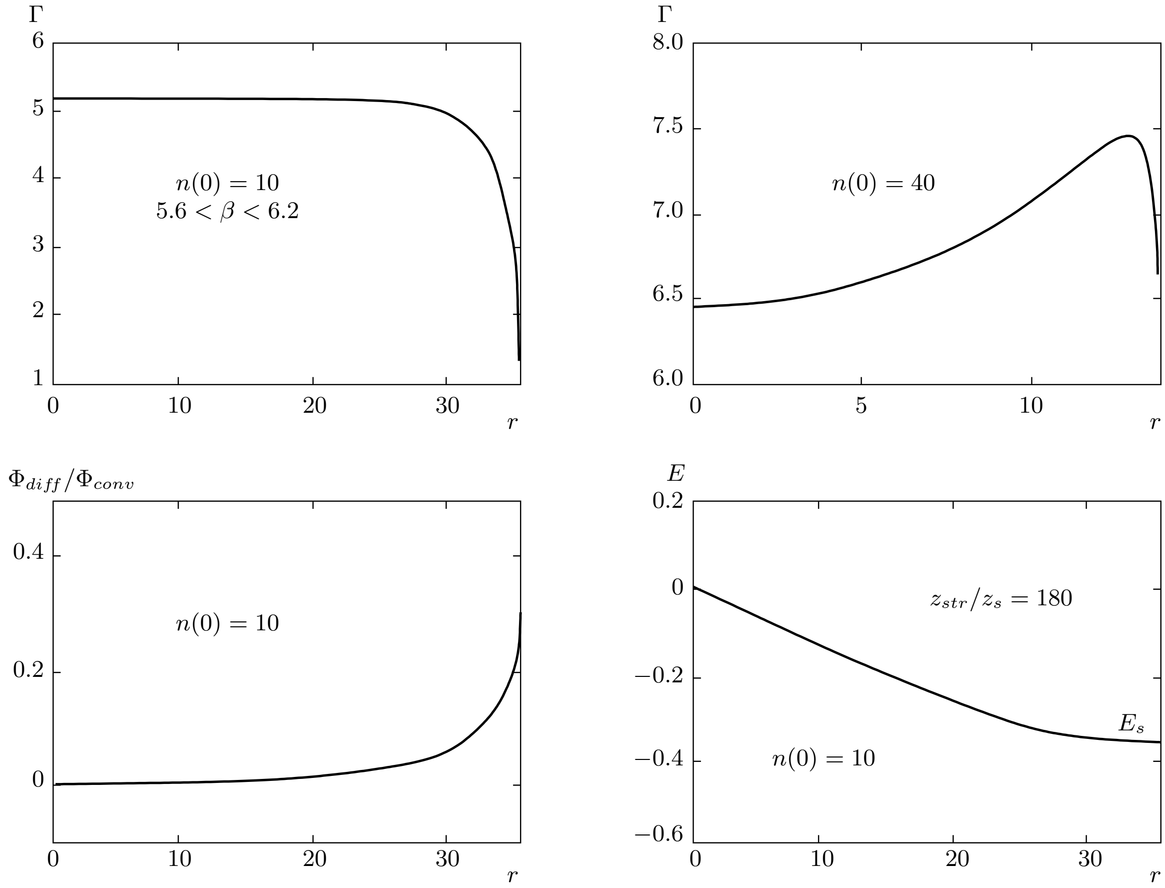

Fig. 2. Dependences of the parameters Γ, the ratio of diffusion to convection flux, and the electric field in the first type of structures on the distance r from their center, $a = 0.01$, $\tau = 0.01$, gas Argon

This is explained by the suppression of diffusion in the regions of high dust density.

3. The applicability of hydrodynamic description (the characteristic length of inhomogeneity is less than the mean free path) has been checked inside the whole structures even at the dust density peaks, which have been specially analyzed with a two-order higher accuracy. This is explained by the shortening of the mean free path in ion dust collisions that dominate at high dust densities, where the peaks appear.

4. The applicability of a continuous description of dust (the characteristic length of inhomogeneities is much larger than the inter-grain distance) is checked to be fulfilled inside all structures.

5. Structures of the first type with a smooth change of dust density up to the edge correspond to the structures recently observed in micro-gravity experiments on board the International Space Station [26].

6. The structures of the third type with several peaks inside them have been observed in recent laboratory experiments [19]. The detailed comparison of the theory and experiments requires additionally taking the specific geometry of experiments into account [19] together with the presence of gravity forces, neglected in present calculations.

7. The master equations take any arbitrary nonlinearity in screening into account and are applicable not only to the models in [21, 22] but also to the nonlinearity related to trapped particles [25].

8. Due to the founded restricted range for the existence of equilibrium of compact structures, it is possible to scan all types of dust structures as a function of only two parameters, the external plasma flux (determining the value $n(0)$) and the ionization rate α_i. Both parameters vary only in restricted ranges and such scanning is practically quite possible in the future, although it requires a large amount of numerical calculations.

9. The two parameters in the calculation correspond to the two existing global parameters that can be changed in experiments, the value of the external

flux and the power of ionization, although it is not easy to establish a one-to-one correspondence between these two sets of parameters because experimentally changing the ionization power also changes the ion density and therefore the flux on the structure surface.

10. For future experiments in spherical chamber, we can recommend to start with rather not large external fluxes because the structures of the first type have the largest size and can confine the largest numbers of grains.

11. Investigations of dust dissipative structures can be one of the best tools for understanding specific properties of self-organization [29, 30].

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